P P SAVANI UNIVERSITY

Third Semester of B. Tech. Examination
May 2019
SESH2031 Differential Methods for Chemical Engineers

16.05.2019, Thursday   Time: 09:00 a.m. To 11:30 a.m.   Maximum Marks: 60

Instructions:
1. The question paper comprises of two sections.
2. Section I and II must be attempted in separate answer sheets.
3. Make suitable assumptions and draw neat figures wherever required.
4. Use of scientific calculator is allowed.

SECTION – I

Q - 1   Answer the following. (Any Five)   [05]

   (i)   For $y_1 = e^x$ & $y_2 = xe^x$ find Wronskian of $y'' - 2y' + y = e^x \log x$.
     a) e^{2x} b) e^{-2x} c) 0   d) 1

   (ii) $L[te^{-t}]$ is
     a) $\frac{1}{s^2}$ b) $\frac{1}{(s+1)^2}$ c) $\frac{1}{(s-1)^2}$ d) $\frac{1}{s}$

   (iii)   The order and degree of the differential equation $\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$ are respectively
     a) $\frac{3}{2}$, 2   b) 2, 3   c) 2, 2   d) 3,4

   (iv)   If $P(x,y)dx + x \sin y \, dy = 0$ is exact then P can be
     a) $\sin y + \cos y$ b) $-\sin y$ c) $x^2 - \cos y$ d) $\cos y$

   (v)   For the differential equation in the form $Pp + Qq = R$ subsidiary equation is
     a) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ b) $\frac{dx}{P} = -\frac{dy}{Q} = \frac{dz}{R}$ c) $\frac{dx}{P^2} = \frac{dy}{Q^2} = \frac{dz}{R^2}$ d) $\frac{dx}{P^2} = -\frac{dy}{Q^2} = \frac{dz}{R^2}$

   (vi)   Solution of the PDE: $z = px + qy + p^2q^2$
     a) $z = ax + by + \sqrt{ab}$ b) $z = ax + by + a^2b^2$
     c) $z = ax + by$ d) None of these

Q - 2 (a)   Answer the following. (Any Two)   [04]

   1. Solve: $\frac{\partial^2 z}{\partial x \, \partial y} = e^{-y} \cos x$
   2. Form the partial differential equation $z = (x-2)^2 + (y-3)^2$.
   3. Solve: $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

(b)   Answer the following (Any Two).   [06]

   1. Define General solution of ODE and solve $(D^3 + 1)y = 0$
   2. Solve PDE: $yp = 2yx + \log q$.
   3. Solve ODE: $\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$

Q - 3   Answer the following. (Any Three)   [05]

   (i)   Solve: $(2x \log x - xy)dy + 2y \, dx = 0$
   (ii)   Find the general solution of $y'' - 2y' - 3y = 6e^{-x} - 8e^x$ using method of undetermined coefficients.
   (iii)   Solve PDE: $4r + 12s + 9t = e^{3x-2y}$.
   (iv)   Find the Laplace transform of $f(t) = \begin{cases} 0 \; ; 0 \le t < 1 \\ t \, ; 1 \le t < 4 \\ 0 \; ; \quad t \ge 4 \end{cases}$

Q - 1 Answer the following. (Any Five) [05]

(i) Product of two even or two odd functions is
 a) even b) odd c) prime d) aliasing

(ii) Z-Transform of k is
 a) $-\frac{kz}{z-1}$ b) $\frac{kz}{z-1}$
 c) $\frac{kz}{z+1}$ d) None of these

(iii) What is $1 + (-1)^n$ when $n = 3$?
 a) 1 b) 0 c) 3 d) -1

(iv) Value of $a + \cos(n\pi)$ when n=4
 a) 1 b) 0 c) 2 d) -1

(v) Which of the following is an even function of t?
 a) t^2 b) $t^2 - 4t$ c) $\sin 2t - 4t$ d) $t^3 + 6$

(vi) Inverse Z- Transform of $\frac{1}{z-a}$ is
 a) a^{n-1} b) a^{n+1} c) $(-a)^{n-1}$ d) $(-a)^{n+1}$

Q - 2 (a) Answer the following. (Any Two) [04]
 1. Define Fourier series expansion for the period $2l$.
 2. Find the Z-transform of $\sin(3n + 5)$
 3. Define Fourier Sine & Cosine Half range series.

Q - 2 (b) Answer the following. (Any Two) [06]
 1. Find the half-range sine series of $f(x) = x^2$ in the interval $(0, \pi)$.
 2. Use convolution theorem to evaluate inverse Z transform of $\left(\frac{z}{z-a}\right)^2$.
 3. Find the Fourier sine transform of f(t) where , $f(t) = \begin{cases} t, 0 \le t \le 1 \\ 0, t > 1 \end{cases}$.

Q - 3 Answer the following. (Any Three) [05]
 (i) Find the Fourier series of $f(x) = x^2$ where $-2 \le x \le 2$.
 (ii) Find the Fourier series to represent $f(x) = e^{ax}$ in the interval $-\pi < x < \pi$.
 (iii) Evaluate Fourier cosine integral of given function $f(x) = e^{-kx}$ where $x > 0, k > 0$.
 (iv) Find Z-transform of following:
 a) $\sin(3n + 5)$
 b) $3n - 4\sin\frac{n\pi}{4} + 5a$.

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